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Differential Configurational Entropy Measurement of Optical Dark Simlaritons

Pooja [a,*], Swarniv Chandra [a,b], Krishna Bulchandani [c], Sukhdeep Kaur [d], Partha Sona Maji [e], Ratbay Myrzakulov [f]

[a] Institute of Natural Sciences and Applied Technology, Kolkata 700032, India
[b] Department of Physics, Government General Degree College, Kushmandi, Dakshin Dinajpur-733121, India
[c] Department of Physics, St. Xavier's College (Autonomous), Mumbai, India
[d] Department of Physics, Guru Nanak Dev University, Amritsar, Punjab, India
[e] Department of Physics, Amity University Kolkata, West Bengal-700135, India
[f] L.N. Gumilev Eurasian National University, Astana 010000, Kazakhstan
* Corresponding Author: poojaphy19@gmail.com

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Abstract: The nonlinear Schrödinger equation (NLSE) describes various types of physical systems such as water waves, nonlinear optics, plasma Physics. In optics, NLSE describes a wide range of non-linearity effects in fiber optics. The solution to the above equation might be examined in order to investigate these consequences. Differential configurational entropy (DCE) is used to analyze the dark similariton solution in this specific situation of NLSE with bright and dark similariton solutions. DCE provides us with the measure of information required to examine stability for various physical systems and to characterize a systems spatial profile using the Fourier transform. It is show that even for the same solitonic solution of the NLS equation, the variations in the spatial shape is induced by different choices of the relevant parameter α. The global minima of the DCE correspond to the saturation of the breadth of dark solitons. While dark similariton waves propagate through waveguides, such low entropic values result in minimum dispersion of momentum modes, and this should be taken into consideration while designing the waveguides.

Keywords: Differential Configurational Entropy, Non-linear Schrödinger equation, Optical Fiber, Dark Simlariton

1. Introduction

The nature of a physical system is of two types, a linear system and a nonlinear system. A nonlinear system is a one which doesn't hold the superposition principle i.e. the response of the system is not proportional to the input it received or in other words sum of two solutions will not give us another solution [1]. In reality almost all the physical systems are of such type. Nonlinear phenomenon is observed in various fields of physics such as dynamical systems, fluid dynamics, and statistical physics and also in chemistry and biology. The behavior of nonlinear systems is described by a set of nonlinear equations, these are the equations which cannot be written as a linear combination of functions or variables. Example of such equations are, the Korteweg-de Vries (KDV) equation that has been used to study waves in shallow water and internal waves in oceans, to investigate acoustic waves propagating across crystal lattices and more recently, also to scrutinize waves in plasma [2] Kadomtsev-Petviashvili (KP) equation, the extension of KDV equation, to study the evolution of long ion-acoustic waves of small amplitude propagating in plasmas under the effect of long transverse perturbations, the NLSE whose domain includes the propagation of light in nonlinear optical fibers [3] and planer waveguides and to Bose-Einstein condensates [4] to the study of fiber optics and formation of rouge waves, The Sine-Gordon equation has many physical applications, including descriptions of chain-like magnetic compounds and transmission lines made out of arrays of Josephson junctions of superconductors [5]. The solution of the above nonlinear equations is generally given by the soliton solution.

Soliton is a self-reinforcing solitary wave packet that maintains its shape while it propagates at a constant velocity. The cancellation of nonlinear and dispersive effects in the medium, a self-reinforcing solitary wave packet is produced which maintains its shape while it propagates at a constant velocity which is termed as soliton. It is a solitary wave that behaves like a ‘particle’, satisfying the two conditions that it must maintain its shape when it moves at constant speed, and when a soliton interacts with another soliton, it emerges from the collision unchanged expect possibly for a phase shift. Research on solitons is quite vibrant in mathematics and physics. In physics, the term soliton and solitary wave are interchangeable. In high energy Physics, solitons are generally known as solution of non-linear field equations whose energy density is localized in space [6]. In optics the term soliton is referred to any optical field that does not change during propagation because of the balance between nonlinear and linear effects in the medium.

Currently proposed extension of Shannon’s information theory applied to spatially-bound or periodic physical systems known as differential configurational entropy (DCE), is a new information tool, to study non-linear physical systems. More precisely, it is a logarithmic measure, in the Fourier space, of the spatial complexity of spatially-bounded functions which represents the exact measure of information that is necessary to describe the spatial shape of functions with respect to their set parameters. This approach has been used to study KDV solitons in quark-gluon plasma in which the optimal width of the solitonic pulse for which the information stored in its spatial shape is most compressed into its momentum modes is derived. The results shows that at a certain value of soliton width for which DCE has a minimum, corresponds to a configuration of maximum compressibility of information in the Fourier modes that describes the spatial shape of the soliton [6].

A Continuous logarithmic measure to study the non-linear physical systems, called as differential configurational entropy (DCE) has been proposed in the recent years. It has been proposed by Gleiser as an extension of information entropy. The differential configurational entropy hence reads,

$$S[f] = -\int_{\infty}^{\infty} \tilde{f}(K) \ln \tilde{f}(K) dK \tag{1}$$

Where, $\tilde{f}(K) = \frac{f(\mathrm{K})}{f_{\max}(K)}$ is the normalized fraction and also makes the DCE to converge [7].

The roots of DCE lie in the study of Shannon entropy. DCE is a new information tool developed to measure spatial complexity in the Fourier space with the given amount of information, introduced to study non-linear physical systems [7]. The basis of DCE lies in the Fourier spectrum of the field configurations which are periodic or are located at definite point in space. For higher DCE, the configuration needs to be more spatially localized which in turn leads to the momentum space to be widespread, on the other hand as the configuration gains uniformity and simplicity, DCE starts decreasing. Each configuration has a specific impression of information in momentum space with measureable complexity. On critical observation of physical systems having local interactions, DCE represents a quantitative measure of entropy of shape. It also allows the exploration of the stability of states in some systems. The roots of the concept of DCE lie in the study of Shannon entropy. In order to understand various factors of complex systems such as of compact astrophysical systems or anti-de sitter black holes, CE is able to effectuate it. The domain of the CE extends from light flavor mesons in dynamical Ads/QCD holographic models to the scalar glue balls, spontaneous symmetry breaking and brane world models. It has been used to investigate the stability of gravitationally bounded stars known as Newtonian polytropes [8].

Gleiser and Stamtopoulos proposed CE as a physical quantity which gives additional information about the systems which have localized energy density. CE of a system with higher energy that provides an approximate solution is high. It has been used to solve the systems with degenerate energies of configurations. The study of CE for various systems has been able to reveal information about non-equilibrium dynamics of spontaneous symmetry breaking and also to investigate the emergence of localized objects during inflationary preheating. The utilization of solitons, Lorentz symmetry breaking was possible with the concept of CE [9].

CE has been applied to several physical systems; a new front in its application is its use in atomic physics, in particular to unstable atomic states. Spontaneous decay of simple one-electron atoms using an information-theoretic approach is examined. Using CE, the average lifetimes of one-electron atoms has been estimated. Still, not much work has been done in this field as it is a recent hot topic and a lot is to be discovered in this area so it becomes of a great interest to various field professionals.

In the present work, we have investigated DCE of non-linear excitation in non-linear physical system. The non-linear Schrödinger equation possess all kind of non-linear excitations such as solitary wave, bright and dark similariton, and rouge waves as the solution. The one studied here is the NLSE with solution of dark similariton. Since DCE forms a versatile framework to formalize uncertainty and predictability so our study extends its application to the non-linear world. The cubic non-linear Schrödinger equation with solution of dark similariton is widely used in optical fiber communications [10], computer networks [11], long-distance telecommunications [12] and sensory receptor cells [13]. It solves the problem of modal dispersion to a considerable extent [14]. Since DCE forms a versatile framework to formalize uncertainty and predictability so our study extends its application to the non-linear world.

We are concerned with the NLSE given as,

$$iU_{\xi} + U_{xx} + \mu |U|^2 U = 0 \tag{2}$$

Where, $u = u(x,\xi)$ is a complex-valued function of two real variables x, ξ and μ is a non-zero real parameter. The physical model of the above written NLSE occur in various areas of physics such as nonlinear optics, water waves, plasma physics, quantum mechanics, superconductivity and Bose-Einstein condensate theory [13, 14]. In optics, the NLSE models many nonlinearity effects in a fiber, including but not limited to self-phase modulation, four-wave mixing, second harmonic generation, stimulated Raman scattering, etc. For water waves, the NLSE describes the evolution of the envelope of modulated nonlinear wave groups. All these physical phenomena can be better understood with the help of exact solutions when they exist for particular values of parameter μ .

It is well known that NLSE admits dark similaritons solution for self-focusing case [13, 14], $\mu > 0$

$$U(x,\xi) = K\sqrt{\frac{-2}{\mu}}\tanh(K(x-2\alpha\xi))Exp(i(\alpha x - (\alpha^2 + 2K^2))\xi) \tag{3}$$

Where, α and k are arbitrary real constants.

The mathematical formulation of differential configurational entropy (DCE) is given as,

$$S_c[\tilde{f}] = -\int_{-\infty}^{\infty} \tilde{f}(x)\ln \tilde{f}(x)dx \tag{4}$$

The position space configurational density is calculated using the Eq.4,

$$\rho(x) = -\frac{K\tanh(K(x-2\alpha\xi))^2}{1+\tanh(2K\alpha\xi)} \qquad (5)$$

The momentum space information density for the wave solution have been obtained using Fourier transform of the corresponding position space wave solution. The momentum space information density $\rho(p)$, using Eq. 4, is obtained as,

$$\rho(p) = \frac{1}{8\pi(1+a_1)^2((1+\tanh[2K\alpha\xi])^2)}((\frac{-1}{p}(p+i(1+a_1))(a_2)a_3+\frac{2a_1}{a_4}(a_4+(1+a_1))a_5 - \frac{1}{a_6}(a_7)(a_7)(a_6-(1+a_1))(a_8)a_9))(-\frac{1}{p}(p-i(1+a_1))(a_8)a_{10}-\frac{1}{a_{11}}a_7(a_{11}-(1+a_1))(a_2a_{12}) \qquad (6)$$

$$a_1 = \exp[4\alpha\xi k],\ a_2 = 2K+ip,\ a_3 = 2F1\left[1,\frac{-ip}{2K},1-\frac{ip}{2K},-a_1\right],\ a_4 = -2iK-p,$$

$$a_5 = 2F1\left[1,1-\frac{ip}{2K},2-\frac{ip}{2K},-a_1\right],\ a_6 = 4K-ip, \%a_7 = \exp[8\alpha\xi k],\ \ a_8 = 2K-ip,$$

$$a_9 = 2F1\left[1,2-\frac{ip}{2K},3-\frac{ip}{2K},-a_1\right],\ a_{10} = 2F1\left[1,\frac{ip}{2K},1+\frac{ip}{2K},-a_1\right],$$

$$a_{11} = 4K+ip,\ a_{12} = 2F1\left[1,2+\frac{ip}{2K},3+\frac{ip}{2K},-a_1\right]$$

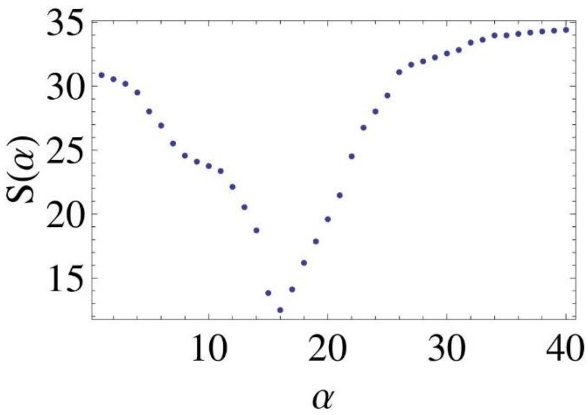

Figure 1. DCE $S(\alpha)$ as a function of the soliton width α. The minimum value of DCE occurs around $\alpha = 16$.

Conclusion

In this work, the informational content of a dark similariton which as a solution from the NLSE is considered. The analysis was based upon the use of the differential configurational entropy (DCE). We showed that the DCE has a minimum for a particular value of the similariton width. It corresponds to a configuration of maximum compressibility of information in the Fourier modes that describes the spatial shape of the similariton. The results show that even for the same similariton solution of the NLSE, the variations in the spatial shape is induced by different choices of the relevant parameter. Our results show that the DCE can be sensitive to variations of the spatial profile for the same solitary wave solution with different values of the parameter. This formalism helps one obtain the optimal width of the dark similariton waves for which dispersion is minimized and the spatial shape is most compressed into its momentum modes. This optimal compression ensures the propagation of minimally entropic dark similariton waves through the waveguide.

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